# Optimal Environmental Taxes Due to Health Effect

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## ABSTRACT

This paper shows that the optimal environmental tax should be less than the marginal environmental damages; since the presence of pre-existing distortionary taxes, increasing the welfare costs is associated with the overall tax code. The reduction of pollution causes the benefits of health by supplying maximum labors to create benefit-side tax interactions, which will tend to reduce the optimal environmental taxes. The paper also shows that the existence of social security system preserves the gross wage of labors during sick days and grants subsidies to medical treatments. An attempt is made to confirm that health effects labor supply, which results in an additional impact, the benefit-side tax-interaction effect.

**JEL. Classification: B22; E62; H21; H23; I11; K23**

**Keywords:** Environmental Pollution, Optimal Environmental Taxes, Public Good, Budget Constraints

## 1 INTRODUCTION

In the recent years the populations of the world is growing rapidly. Larger parts of this population are living in the South Asia. China, India, Pakistan, Bangladesh, Myanmar, Indonesia and some other countries of the South Asia are densely populated. For the daily needs of residence and cultivation human is continually destroying forests and wetlands. On the other hand, various industries are growing rapidly in every country of the world for the demand of the large population. As a result, the environmental pollution is increasing which effects on human health.

Tullock (1967) and Terkla (1984) were the first who suggested that revenues from environmental taxation could be used to finance reductions in pre-existing taxes. We see that this fund recycling process would significantly reduce the welfare costs associated with the overall tax code. However, it did not account for the effects of health damages from pollution. The paper of Schwartz and Repetto (2000) suggested that such health effects may cause the optimal environmental tax to exceed the marginal damage from pollution. The revenue got from environmental taxation was used to reduce taxation of labor; it would reduce the distortion in the labor market. Hence, it would be used to reduce the deadweight loss associated with income taxes. As

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a result, the double dividend hypothesis implies that the optimal environmental tax generally exceeds marginal damages when revenues are used in this way. We show as like Williams (2003) and Caffet (2005) that this is not the case. The papers of 1990s, Bovenberg and de Mooij (1994), Bovenberg and Van der Ploeg (1994, 1996), Parry (1995), Goulder (1995), Goulder; Parry; Williams III, and Burtraw (1999), and other authors have demonstrated that the existence of an offsetting tax-interaction effect. Pollution taxes drive up the price of consumption goods, so that lower the real wage, causing households to work less and consume more leisure which raise the environmental taxes.

We see that in the papers of 1990s have cited above suggested that the ability to simultaneously curb pollution and tax distortions are much more limited than previously thought. They emphasized on the second-best negative impacts of pollution taxes. Fullerton and West (2010), Islam; Mohajan and Paul (2011) have discussed the taxes on cars and gasoline. They have shown that if government is applied a tax for engine size and gasoline, and the motorists obey the rules accordingly then environment pollution must be decreased. Blundell and Shephard (2011) recently have analyzed in detail about optimal taxation of low-income families, which will be beneficial to the family and to the government. Bossi; Calcott and Petkov (2011) have studied the implementation of the social optimum in a model of habit formation. They have considered taxes that address inefficiencies due to negative consumption externalities, imperfect competition, and self-control problems. David and Sinclair-Desgagn´e (2010) and Hattori (2011) characterized optimal environmental policy in a case where innovation in clean production technologies is developed and provided by a monopoly. Fullerton (2011) has used the example of a proposed carbon permit system to illustrate and has discussed six different types of distributional effects. Himmes and Weber (2011) analyzed the optimal policy design in a context with more than one externality while taking explicitly into account uncertainty surrounding future emission damage costs. Kaplow (2011) has given the technique to illuminate and has extended Tax by Design’s analysis regarding the VAT, environmental taxation, wealth transfer taxation, and income transfers. Mohajan, Deb and Chakraborty (2011) show that due to environment pollution workers suffer from illness; as a result economic development decreases. Ploeg and Withagen (2010, 2011a,b) have analyzed the optimal time of transition from fossil fuel to renewable, amount of fossil fuel to leave in situ, and carbon tax. Ulph and Ulph (2011) have discussed the optimal design of climate change policies when a government wants to encourage the private sector to undertake significant immediate investment in developing cleaner technologies, but the relevant carbon taxes or other environmental policies that would incentives such investment by firms will be set in the future.

In this paper we analysis the effect of health from environment pollution in the light of the above mention papers. The model confirms that health effects labor supply, which results in an additional impact, the benefit-side tax-interaction effect.

The paper is organized as follows: In section-2, we discuss the mathematical model of environmental tax. This is used to discuss the beneficial side of environmental tax in section-3 and optimal environmental tax in section-4. Final section-5 is of concluding remarks. Derivation of equations (16) and (17) are given in appendices.

## 2 THE MODEL

Let us consider a representative agent. The households divide the endowment time $T$ between leisure $l$ and labor $L$, hence we can write;

$$T = l + L. \tag{1}$$

At the time $T$ the agent produce the two consumption goods $X$ and $Y$, where $X$ is polluted good but $Y$ is clean good. Now the maximized utility function $U$ of the households can be written as (Williams 2003 and Caffet 2005):

$$U = U(X, Y, l, H, G) \tag{2}$$

which is continuous and quasi-concave. Here $H$ is the consumer health, and $G$ is the quality of a public good.

The health situation of consumer is a function of both environmental quality $Q$ and medical expenditure $M$, so that;

$$H = H(M, Q). \tag{3}$$

By the practical experience it can be written as;

$$\frac{\partial H}{\partial M} > 0, \frac{\partial H}{\partial Q} > 0, \text{ and } \frac{\partial^2 H}{\partial M^2} < 0.$$

In the model as like Caffet (2005) we have considered consumption equals production, so that they are equivalent. Good $X$ is dirty, cause a negative externality, either in consumption or in production but good $Y$ is clean and hence no such externality. Consumption of polluted good $X$ has two effects:

- $X$ diminishes the consumer welfare by deteriorating his health condition.
- $X$ causes households to lose some period to sickness.

For taking polluted good households lose some period to sickness. As a result their endowment time $T$ will be reduced so that they lose their total allocated time for work or leisure or for both i.e., they face a household time constraint as follows:

$$L + l < T. \tag{4}$$

Williams (2003) has considered the sickness $S$ is a function of environmental quality $Q$, i.e.,

$$S = S(Q),$$

But Caffet (2005) has considered that the sick days are not excessively a function of environmental quality but more generally of the agent's health condition i.e.,

$$S = S(H), \text{ with } \frac{\partial S}{\partial H} < 0. \tag{5}$$

We see that Caffet's assumption is more realistic, since it will allow us to better approach interactions between subsidies to medical expenditures and environmental taxation. By this assumption the household time constraint can be written as;

$$L + l = T - S(H). \tag{6}$$

To normalize as like Caffet (2005) we consider for pollution and consumption of one unit of the polluting good $X$ emits one unit of pollution. Hence, an exogenous baseline level of environmental quality $\overline{Q}$ will be related with environmental quality $Q$ as follows:

$$Q = \overline{Q} - X. \tag{7}$$

Again $X$, $Y$, $G$ and $M$ are all produced using labor as the only factor of production, so that all of them are considered as goods. In the model units are normalized such that one unit of labor can produce one unit of any of the four goods, hence;

$$f(L) = L = X + Y + G + H. \tag{8}$$

The government or social security system imposes a tax $\tau_X$ on good $X$, and also include a tax on labor income $\tau_L$. The government or social security system preserves the gross wage during sick-days, and grants subsidies to medical expenditures. In the model as like Caffet (2005) we assume a global subvention rate $\tau_M$. For simplicity we also normalize the gross wage to equal one, and then the consumer budget constraint can be written as follows:

$$(1-\tau_L)(L+S) + I = (1+\tau_X)X + Y + (1-\tau_M)M \tag{9}$$

Where $I$ is lump-sum income, which is assumed to be zero. In the budget constraint we always include $I$, even though households do not have any lump-sum income, in order to provide a strong expression for income effects.

For simplicity we assume that the tax revenue is used to provide a fixed quantity of the public good;

$$G = \tau_L L + \tau_X X. \tag{10}$$

Government income is the sum of labor and pollution tax revenues, hence it can be written as follows:

$$\tau_L(L+S) + \tau_X X = G + \tau_M M + S. \tag{11}$$

The household maximum utility (2) subject to their time (6) and budget constraints (9), taking the quantity of the public good (10), the tax rates and the level of environmental quality as given. Incorporating all the information mentioned above and after some simplification, we can write the maximum utility function (2) as follows:

$$U = C + \lambda\left(Y + (1+\tau_X)X + (1-\tau_L)l + (1-\tau_M)M\right) \tag{12}$$

Where $\lambda$ is the marginal utility of income and $C$ is for constant terms.

Applying the first order partial differentiation of (12) for maximization, we obtain:

$$\frac{\partial U}{\partial Y} = \lambda , \qquad (13a)$$

$$\frac{\partial U}{\partial X} = \lambda (1 + \tau_X), \qquad (13b)$$

$$\frac{\partial U}{\partial l} = \lambda (1 - \tau_L), \qquad (13c)$$

$$\frac{\partial U}{\partial H} = \lambda (1 - \tau_M) \frac{\partial M}{\partial H} . \qquad (13d)$$

These first order conditions, together with other equations given so far implicitly define the uncompensated demand functions as follows:

$$X(\tau_X, \tau_L, \tau_M, Q, I); \quad Y(\tau_X, \tau_L, \tau_M, Q, I); \quad l(\tau_X, \tau_L, \tau_M, Q, I); \quad M(\tau_X, \tau_L, \tau_M, Q, I)$$

## 3 BENEFICIAL SIDES OF ENVIRONMENTAL TAX

In this section, we will examine the impacts of welfare for imposing environmental tax. First, we define key-parameter following Bovenberg (1999), Williams (2003) and Caffet (2005) as follows:

$$\eta = \frac{\tau_L \frac{\partial l}{\partial \tau_L}}{L + S(H) - \tau_L \frac{\partial l}{\partial \tau_L}} + 1 \qquad (14)$$

This is the marginal cost of public funds (MCPF), the cost to the household of raising a marginal dollar of government revenue through the labor tax. The denominator of (14) is the marginal tax revenue for an incremental increase in the labor tax rate, $\tau_L$ which can be obtained by differentiating the term $\tau_L (L + S)$. The numerator of (14) is the welfare loss from an incremental increase in $\tau_L$ which is the wedge between the gross wage and the net wage, and multiplied by the reduction in labor supply. Practical experience suggests that while male labor supply may be backward bending, that is estimates of the male labor-supply elasticity are close to zero, female labor-supply elasticity estimates are much higher and thus total labor supply is definitely not backward bending (Fuchs, Krueger and Poterba 1998). Here gross wage equals to the value marginal product of labor and the net wage equals to the marginal social cost of labor in terms of foregone labor. The first term of (14) is the marginal deadweight loss per dollar of revenue. The cost to households is the deadweight loss plus the revenue, so that MCPF equals first term plus one. This is a partial equilibrium definition of the MCPF, since it ignores all effects outside the labor market, including changes in the deadweight loss in the market for good *X*, environmental quality and the revenue from the corrective tax. The marginal revenue from the labor tax is positive so that from (14) we see that $\eta > 1$.

Williams (2003) defines marginal damages from pollution $\tau_P$ as the sum of two terms, the respective values of the direct utility loss from reduced health and the time lost to illness where the gross wage has been normalized to equal one, as follows:

$$\tau_P = \frac{1}{\lambda}\frac{\partial U}{\partial H}\frac{\partial H}{\partial Q} - \frac{\partial S}{\partial H}\frac{\partial H}{\partial Q}.$$

Since the representative agent still earns his wage during sick-days, so that Caffet (2005) corrected the second term of it. Caffet's logic is that households only bear the distortionary cost introduced by the government transfers, which is the marginal deadweight loss of the labor tax relative to the wage keeping device. In this way, the Pigovian rate is given by:

$$\tau_P = \frac{1}{\lambda}\frac{\partial U}{\partial H}\frac{\partial H}{\partial Q} - (\eta - 1)\frac{\partial S}{\partial H}\frac{\partial H}{\partial Q}. \tag{15}$$

With the basis of (15), the welfare effect of the environmental tax being as follows (for derivation see Appendix-I):

$$\frac{1}{\lambda}\frac{dU}{d\tau_X} = \underbrace{(\tau_X - \tau_P)\frac{dX}{d\tau_X}}_{dW^P} + \underbrace{(\eta - 1)\left(X + \tau_X\frac{dX}{d\tau_X}\right)}_{dW^R} - \underbrace{\eta\left[\tau_L\frac{\partial l}{\partial \tau_X} + \left(\tau_M + \frac{\partial S}{\partial H}\frac{\partial H}{\partial M}\right)\frac{dM}{d\tau_X}\right]}_{dW^I}$$

$$- \underbrace{\left[\frac{\partial S}{\partial H}\frac{\partial H}{\partial Q} + \eta\tau_L\frac{\partial l}{\partial Q}\right]\frac{dQ}{d\tau_X}}_{dW^{IB}} \tag{16}$$

The 1st term of (16) is the primary or Pigovian effect, $dW^P$ which is the effect of the tax on the pollution externality. In a first-best world without pre-existing distortions, the optimum is reached for $\tau_X = \tau_P$, which means that the optimal environmental tax must equal to the Pigovian rate.

The 2nd term is the gain from the marginal revenue-recycling effect, $dW^R$, which is the welfare gain from using the pollution tax revenues to reduce the labor tax. This is happen when they are returned lump-sum and have no efficiency consequences at all. So that $dW^R$ is the product of the efficiency value per dollar of revenue and the incremental pollution tax revenue, where the marginal excess burden of taxation.

The 3rd term $dW^I$ and 4th term $dW^{IB}$ are respectively what Williams (2003) called cost-side and benefit-side tax interaction effects. They activate when changes in households' labor supply decisions interact with the labor market distortion.

For the 3rd term, the pollution tax drives up the price of consumer polluting goods, which lower the real wage and as a result it affects discouraging labor supply. We have seen that this fall in labor supply must exacerbate the private social cost of the labor tax by $\tau_L \frac{\partial l}{\partial \tau_X}$ (Caffet 2005). The beneficial side is that this also reduces labor tax revenues equal to $\tau_L \frac{\partial l}{\partial \tau_X}$ and thus requiring a compensating increase in the labor tax rate and creating an efficiency loss which equals to the amount that the government has to refund multiplied by the efficiency cost per dollar of labor tax revenue, that is, the marginal excess burden of labor taxation. The tax interaction effect identified by prior literature on the double dividend hypothesis is exclusively the welfare loss from these two impacts, which equals to $\eta \tau_L \frac{\partial l}{\partial \tau_X}$. In the analysis of Caffet (2005), there is however a second term in $dW^I$. In this term, first part indicates medical expenditures generate an efficiency gain, $(\eta - 1)\tau_M \frac{\partial X}{\partial \tau_X}$ which is related to the contraction of subsidies' amount. Second part indicates subventions cause this consumption to be under priced relative to its social cost. As a result, any decrease in medical expenditures will lead to a general-equilibrium welfare gain. This gain equals the wedge between supply and demand prices multiplied by the reduction of subsidized consumption, which is equals to $\tau_M \frac{\partial X}{\partial \tau_X}$.

The 4th term $dW^{IB}$ is the benefit-side tax interaction effects, which expresses the impact of improved environmental quality on labor supply decisions. Caffet's logic is that a rise in sick-days does not imply a decrease in government revenues. Because $\tau_L$ is levied on $L + S$ instead of effective labor income $L$. So that welfare impacts of sick-days reduction can be resumed by the product of that decrease and the sum of the efficiency gain and the private social benefit relative to that decrease.

## 4 OPTIMAL ENVIRONMENTAL TAX

Following Williams (2003), we now stress on the two opposite tax interaction effects, $dW^I$ and $dW^{IB}$. Following Caffet (2005) we also consider the neutral assumption that goods $X$ and $Y$ are equal substitutes for leisure, allowing equation (16) to be re-written as follows (for derivation see APPENDIX-II);

$$\frac{1}{\lambda}\frac{dU}{d\tau_X} = (\eta\tau_X - \tau_P)\frac{dX}{d\tau_X} - \eta\left(\tau_M + \frac{\partial S}{\partial H}\frac{\partial H}{\partial M}\right)\frac{dM}{d\tau_X} + \left\{-\frac{\partial S}{\partial H}\frac{\partial H}{\partial Q} - (\eta - 1)\frac{\in_{LI}}{\in_L}\left[1 - (1 - \tau_M)\frac{\partial M}{\partial I}\right]^{-1}(1 - \tau_M)\frac{\partial M}{\partial Q}\right\}\frac{dQ}{d\tau_X} \tag{17}$$

Where $\in_L$ is the uncompensated labor supply elasticity and $\in_{LI}$ is the income elasticity of labor supply.

The first term on the right hand side combines the primary welfare effect, the first component of cost-side tax interaction effect and the revenue-recycling effect. Without the other terms on the right hand side, it indicates that the optimal environmental tax is equal to the Pigovian rate divided by the MCPF. The second term symbolizes the interactions have already identified between subsidies to medical expenditures and environmental taxation. Finally, the last term shows that benefit-side tax interaction effects must improved health conditions, may significantly reduce the gross cost of an environmental tax reform.

Health effects might lead to a double dividend if an environmental tax on both reduces pollution and raise welfare, which is exclusive of the environmental benefits (Goulder 1995). If the second dividend is defined as a welfare gain exclusive of both direct and indirect benefits of reduction pollution, then excluding both the primary effect and the benefit-side tax-interaction effect, the introducing health effects will not produce a double dividend. Even if double dividend only deals with an additional hypothesis, we have tried to show that it is a crucial point in studies modeling health effects in the agent's utility function (Caffet 2005). By introducing labor market imperfections in models which deal with the possibility of finding labor market imperfections in models which deal with the possibility of obtaining a second dividend in the form of a reduced unemployment level. This assumes the existence of a social security system seems that it is necessary to correctly estimate double dividend prospects in explicitly modeling health studies (Bovenberg and Van der Ploeg 1994, Caffet 2005).

Williams (2003) used typical parameters, a labor tax $\tau_L$ = 0.4, an uncompensated labor supply elasticity $\in_L$ = 0.15 and an income elasticity of labor supply $\in_{LI} = -0.15$ and obtained the last term of his equation (11) is actually negative. In the model of Caffet (2005) and in our model for the same typical parameters, the sign of benefit-side tax interaction effects is undetermined, since the term $-\frac{\partial S}{\partial H}\frac{\partial H}{\partial Q}$ is positive, which represents the welfare gain due to the reduction of social transfer relative to the wage-keeping device.

Finally we can say with Caffet (2005) that if the negative income effect due to the reduction of transfers $S(H)$ is superior to the positive income effect due to the reduction of medical expenditures, then the substitution of $S(H)$ by $L$ implies that the optimal environmental tax rate is greater than traditionally highlighted by the models of Bovenberg (1999); Bovenberg and de Mooij (1994), Bovenberg and Goulder (1996), Bovenberg and Ploeg (1994; 1996), Parry (1995), Goulder (1995), Goulder; Parry; Williams III, and Burtraw.(1999), Blundell and Shephard (2011), Bossi ;Calcott and Petkov (2011), David and Sinclair-Desgagn´e (2010), Fullerton and West (2010), Fullerton (2011), Hattori (2011), Himmes and Weber (2011), Kaplow(2011), and Ploeg and Withagen (2010, 2011a,b).

## 5 CONCLUDING REMARKS

This paper shows that optimal environmental tax rate might exceed marginal damages. Bovenberg (1999); Bovenberg and Mooij (1994), Bovenberg and Goulder (1996), Bovenberg and Ploeg (1994), Parry (1995), Goulder (1995) or Goulder; Parry; Williams III, and Burtraw (1999) have analyzed the double dividend hypothesis. Schwartz and Repetto (2000) suggest that health damages from pollution may cause the optimal environmental tax to exceed the marginal damage from pollution. But we show that actually this is not the case. The social security system tends to magnify benefit-side tax interactions, which could finally offset cost side ones. This paper is prepared mainly following Caffet (2005) and Williams (2003). We have tried to give mathematical calculations and physical interpretations in some details. Derivation of equations (16) and (17) are given in Appendix with detail calculations thinking for the readers who are new in this field. We hope that the readers will feel no bother when study this paper and realize the importance of the optimal environmental taxes due to health effects.

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## APPENDIX-I

**Derivation of equation (16):**

Taking the total derivative of utility (12) with respect to the corrective tax $\tau_X$, substituting the consumer first-order conditions (13), and for constant public spending ($dG = 0$), and finally dividing both sides by $\lambda$ we obtain as follows:

$$\frac{1}{\lambda}\frac{dU}{d\tau_X}=\frac{dY}{d\tau_X}+\left(1+\tau_X\right)\frac{dX}{d\tau_X}+\left(1-\tau_L\right)\frac{dl}{d\tau_X}+\left(1-\tau_M\right)\frac{dM}{d\tau_X}+\frac{1}{\lambda}\frac{\partial U}{\partial H}\frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X}. \qquad \text{(AI-1)}$$

Taking a total derivative of the production function (8) with respect to $\tau_X$ gives;

$$\frac{dL}{d\tau_X}=\frac{dY}{d\tau_X}+\frac{dX}{d\tau_X}+\frac{dG}{d\tau_X}+\frac{dM}{d\tau_X}. \qquad \text{(AI-2)}$$

Again taking the total derivative of the household time constraint (6) we obtain;

$$\frac{dL}{d\tau_X}+\frac{dl}{d\tau_X}=\frac{dT}{d\tau_X}-\frac{\partial S}{\partial H}\left(\frac{\partial H}{\partial M}\frac{dM}{d\tau_X}+\frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X}\right). \qquad \text{(AI-3)}$$

Further consider the household's time endowment $T$ and the public good $G$ be constant, so that using $dG=0$ and $dT=0$, then from (AI-2) and (AI-3) we get:

$$\frac{dL}{d\tau_X}=\frac{dY}{d\tau_X}+\frac{dX}{d\tau_X}+\frac{dM}{d\tau_X}, \qquad \text{(AI-4)}$$

$$\frac{dL}{d\tau_X}+\frac{dl}{d\tau_X}=-\frac{\partial S}{\partial H}\left(\frac{\partial H}{\partial M}\frac{dM}{d\tau_X}+\frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X}\right). \qquad \text{(AI-5)}$$

From (AI-4) and (AI-5) takes we can write;

$$\frac{dY}{d\tau_X}+\frac{dX}{d\tau_X}+\frac{dM}{d\tau_X}+\frac{dl}{d\tau_X}=-\frac{\partial S}{\partial H}\left(\frac{\partial H}{\partial M}\frac{dM}{d\tau_X}+\frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X}\right). \qquad \text{(AI-6)}$$

Using (AI-6) in (AI-1) can be written as;

$$\frac{1}{\lambda}\frac{dU}{d\tau_X}=\tau_X\frac{dX}{d\tau_X}-\tau_L\frac{dl}{d\tau_X}-\tau_M\frac{dM}{d\tau_X}+\frac{1}{\lambda}\frac{\partial U}{\partial H}\frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X}-\frac{\partial S}{\partial H}\left(\frac{\partial H}{\partial M}\frac{dM}{d\tau_X}+\frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X}\right). \qquad \text{(AI-7)}$$

From (15) we obtain;

$$\tau_P + (\eta - 1)\frac{\partial S}{\partial H}\frac{\partial H}{\partial Q} = \frac{1}{\lambda}\frac{\partial U}{\partial H}\frac{\partial H}{\partial Q}. \qquad \text{(AI-8)}$$

Using (AI-8) in (AI-7) can be written as follows:

$$\frac{1}{\lambda}\frac{dU}{d\tau_X} = \tau_X \frac{dX}{d\tau_X} + \tau_P \frac{dQ}{d\tau_X} + (\eta - 1)\frac{\partial S}{\partial H}\frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X} - \tau_L \frac{dl}{d\tau_X} - \tau_M \frac{dM}{d\tau_X} - \frac{\partial S}{\partial H}\left(\frac{\partial H}{\partial M}\frac{dM}{d\tau_X} + \frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X}\right). \qquad \text{(AI-9)}$$

Using $dQ = -dX$ only in second term of right hand side of (AI-9) we get;

$$\frac{1}{\lambda}\frac{dU}{d\tau_X} = (\tau_X - \tau_P)\frac{dX}{d\tau_X} + (\eta - 2)\frac{\partial S}{\partial H}\frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X} - \tau_L \frac{dl}{d\tau_X} - \left(\tau_M + \frac{\partial S}{\partial H}\frac{\partial H}{\partial M}\right)\frac{dM}{d\tau_X}. \qquad \text{(AI-10)}$$

We now take steps to make the term $\frac{dl}{d\tau_X}$ more explicit. This term symbolizes interactions between the environmental tax and household labor supply decisions. According to the uncompensated demand equations and since the government adjusts its budget constraint with $\tau_L$ (i.e., $\frac{d\tau_L}{d\tau_X} = 0$), we can write it as follows:

$$\frac{dl}{d\tau_X} = \frac{\partial l}{\partial \tau_X} + \frac{\partial l}{\partial \tau_L}\frac{d\tau_L}{d\tau_X} + \frac{\partial l}{\partial Q}\frac{dQ}{d\tau_X}. \qquad \text{(AI-11)}$$

Taking the total derivative of (9) we get;

$$\frac{dL}{d\tau_X} = -\frac{dl}{d\tau_X} - \frac{\partial S}{\partial H}\left(\frac{\partial H}{\partial M}\frac{dM}{d\tau_X} + \frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X}\right). \qquad \text{(AI-12)}$$

Taking the total derivative of (11) we obtain;

$$(L + S)\frac{d\tau_L}{d\tau_X} + \tau_L\left[\frac{dL}{d\tau_X} + \frac{\partial S}{\partial H}\left(\frac{\partial H}{\partial M}\frac{dM}{d\tau_X} + \frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X}\right)\right] + \tau_X \frac{dX}{d\tau_X} + X = \tau_M \frac{dM}{d\tau_X}$$

$$+ \frac{\partial S}{\partial H}\left(\frac{\partial H}{\partial M}\frac{dM}{d\tau_X} + \frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X}\right),$$

$$-(L+S)\frac{d\tau_L}{d\tau_X}=\tau_L\frac{dL}{d\tau_X}-(1-\tau_L)\frac{\partial S}{\partial H}\left(\frac{\partial H}{\partial M}\frac{dM}{d\tau_X}+\frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X}\right)+\tau_X\frac{dX}{d\tau_X}+X-\tau_M\frac{dM}{d\tau_X}$$

.

(AI-13)

Using (AI-12) in (AI-13) yield:

$$\frac{d\tau_L}{d\tau_X}=-\frac{1}{L+S}\left[X+\tau_X\frac{dX}{d\tau_X}-\tau_M\frac{dM}{d\tau_X}-\tau_L\frac{dl}{d\tau_X}-\frac{\partial S}{\partial H}\left(\frac{\partial H}{\partial M}\frac{dM}{d\tau_X}+\frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X}\right)\right],$$

$$\frac{d\tau_L}{d\tau_X}=\tau_L\frac{1}{L+S}\frac{dl}{d\tau_X}-\frac{1}{L+S}\left[X+\tau_X\frac{dX}{d\tau_X}-\tau_M\frac{dM}{d\tau_X}-\frac{\partial S}{\partial H}\left(\frac{\partial H}{\partial M}\frac{dM}{d\tau_X}+\frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X}\right)\right].$$

(AI-14)

This expression gives the reduction in labor tax that can be financed by a marginal increase in the environmental tax, while maintaining budget balance.

Using (AI-14) in (AI-11) we find:

$$\frac{dl}{d\tau_X}=\frac{\partial l}{\partial\tau_X}+\tau_L\frac{1}{L+S}\frac{\partial l}{\partial\tau_L}\frac{dl}{d\tau_X}-\frac{1}{L+S}\frac{\partial l}{\partial\tau_L}\left[X+\tau_X\frac{dX}{d\tau_X}-\tau_M\frac{dM}{d\tau_X}-\frac{\partial S}{\partial H}\left(\frac{\partial H}{\partial M}\frac{dM}{d\tau_X}+\frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X}\right)\right]$$

$$+\frac{\partial l}{\partial Q}\frac{dQ}{d\tau_X},$$

$$(L+S)\frac{dl}{d\tau_X}=(L+S)\frac{\partial l}{\partial\tau_X}+\tau_L\frac{\partial l}{\partial\tau_L}\frac{dl}{d\tau_X}-\frac{\partial l}{\partial\tau_L}\left[X+\tau_X\frac{dX}{d\tau_X}-\tau_M\frac{dM}{d\tau_X}-\frac{\partial S}{\partial H}\left(\frac{\partial H}{\partial M}\frac{dM}{d\tau_X}+\frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X}\right)\right]$$

$$+(L+S)\frac{\partial l}{\partial Q}\frac{dQ}{d\tau_X},$$

$$(L+S)\frac{dl}{d\tau_X}-\tau_L\frac{\partial l}{\partial\tau_L}\frac{dl}{d\tau_X}=(L+S)\frac{\partial l}{\partial\tau_X}-\frac{\partial l}{\partial\tau_L}\left[X+\tau_X\frac{dX}{d\tau_X}-\tau_M\frac{dM}{d\tau_X}-\frac{\partial S}{\partial H}\left(\frac{\partial H}{\partial M}\frac{dM}{d\tau_X}+\frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X}\right)\right]$$

$$+\left(L+S\right)\frac{\partial l}{\partial Q}\frac{dQ}{d\tau_X},$$

$$\frac{dl}{d\tau_X}=\frac{1}{\left(L+S\right)-\tau_L\frac{\partial l}{\partial \tau_L}}\left\{\left(L+S\right)\frac{\partial l}{\partial \tau_X}-\frac{\partial l}{\partial \tau_L}\left[X+\tau_X\frac{dX}{d\tau_X}-\tau_M\frac{dM}{d\tau_X}-\frac{\partial S}{\partial H}\left(\frac{\partial H}{\partial M}\frac{dM}{d\tau_X}+\frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X}\right)\right]\right.$$

$$\left.+\left(L+S\right)\frac{\partial l}{\partial Q}\frac{dQ}{d\tau_X}\right\}. \qquad \text{(AI-15)}$$

From (14) we can write;

$$L+S\left(H\right)-\tau_L\frac{\partial l}{\partial \tau_L}=\frac{\tau_L\frac{\partial l}{\partial \tau_L}}{\eta-1} \quad \text{and} \quad L+S\left(H\right)=\frac{\eta}{\eta-1}\tau_L\frac{\partial l}{\partial \tau_L},$$

then (AI-15) becomes;

$$\frac{dl}{d\tau_X}=\frac{1}{\frac{1}{\eta-1}\tau_L\frac{\partial l}{\partial \tau_L}}\left\{\frac{\eta}{\eta-1}\tau_L\frac{\partial l}{\partial \tau_L}\frac{\partial l}{\partial \tau_X}-\frac{\partial l}{\partial \tau_L}\left[X+\tau_X\frac{dX}{d\tau_X}-\tau_M\frac{dM}{d\tau_X}-\frac{\partial S}{\partial H}\left(\frac{\partial H}{\partial M}\frac{dM}{d\tau_X}+\frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X}\right)\right]\right.$$

$$\left.+\frac{\eta}{\eta-1}\tau_L\frac{\partial l}{\partial \tau_L}\frac{\partial l}{\partial Q}\frac{dQ}{d\tau_X}\right\}. \qquad \text{(AI-16)}$$

Using (AI-16) in (AI-10) the equation (AI-17) becomes as follows:

$$\frac{1}{\lambda}\frac{dU}{d\tau_X}=\left(\tau_X-\tau_P\right)\frac{dX}{d\tau_X}+\left(\eta-1\right)\left(X+\tau_X\frac{dX}{d\tau_X}\right)-\left(\eta-1\right)\left(\tau_M+\frac{\partial S}{\partial H}\frac{\partial H}{\partial M}\right)\frac{dM}{d\tau_X}-\eta\tau_L\frac{\partial l}{\partial \tau_X}$$

$$-\left(\eta-1\right)\frac{\partial S}{\partial H}\frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X}-\left(\tau_M+\frac{\partial S}{\partial H}\frac{\partial H}{\partial M}\right)\frac{dM}{d\tau_X}-\left(\eta-2\right)\frac{\partial S}{\partial H}\frac{\partial H}{\partial Q}\frac{dQ}{d\tau_X},$$

$$\frac{1}{\lambda}\frac{dU}{d\tau_X}=\left(\tau_X-\tau_P\right)\frac{dX}{d\tau_X}+\left(\eta-1\right)\left(X+\tau_X\frac{dX}{d\tau_X}\right)-\eta\left[\tau_L\frac{\partial l}{\partial \tau_X}+\left(\tau_M+\frac{\partial S}{\partial H}\frac{\partial H}{\partial M}\right)\frac{dM}{d\tau_X}\right]$$

$$-\left[\frac{\partial S}{\partial H}\frac{\partial H}{\partial Q}+\eta\tau_L\frac{\partial l}{\partial Q}\right]\frac{dQ}{d\tau_X} \ . \qquad \text{(AI-17)}$$

This is the required equation (16).

**APPENDIX-II**

**Derivation of equation (17):**

Assume that utility, the levels of public spending and environmental quality be constant. Now taking the total derivative of utility (12) with respect to $\tau_L$ we get;

$$\frac{\partial l^c}{\partial \tau_L}=-\frac{\partial l^c}{\partial(1-\tau_L)}$$

$$=\frac{\frac{\partial U}{\partial X}}{\frac{\partial U}{\partial l}}\frac{\partial X^c}{\partial(1-\tau_L)}+\frac{\frac{\partial U}{\partial Y}}{\frac{\partial U}{\partial l}}\frac{\partial Y^c}{\partial(1-\tau_L)}+\frac{\frac{\partial U}{\partial H}}{\frac{\partial U}{\partial l}}\frac{\partial M^c}{\partial(1-\tau_L)}$$

$$=\frac{1+\tau_X}{1-\tau_L}\frac{\partial X^c}{\partial(1-\tau_L)}+\frac{1}{1-\tau_L}\frac{\partial Y^c}{\partial(1-\tau_L)}+\frac{1-\tau_M}{1-\tau_L}\frac{\partial M^c}{\partial(1-\tau_L)}, \qquad \text{(AII-1)}$$

where '*c*' denotes a compensated derivative.

The assumption that the cross elasticity of *X* and leisure is equal to the average (weighted by assumption shares) over all goods is given by:

$$\in_{X,I}=S_X\in_{X,I}+S_Y\in_{Y,I}+S_M\in_{M,I}\,, \qquad \text{(AII-2)}$$

where $\in_{i,I}=\dfrac{\partial i^C}{\partial(1-\tau_L)}\dfrac{1-\tau_L}{i}$ and $s_i=\dfrac{p_i i}{\sum(p_i i)}$ respectively represent the compensated elasticity of demand for good *i* with respect to the price of leisure and share of that good *i* in total consumption. Here *i* = *X*, *Y*, *M*.

Now (AII-2) can be written as follows:

$$\frac{\partial X^C}{\partial(1-\tau_L)}\frac{1-\tau_L}{X}=\frac{p_X X}{\sum p_X X}\frac{\partial X^C}{\partial(1-\tau_L)}\frac{1-\tau_L}{X}+\frac{p_Y Y}{\sum p_Y Y}\frac{\partial Y^C}{\partial(1-\tau_L)}\frac{1-\tau_L}{Y}+\frac{p_M M}{\sum p_M M}\frac{\partial M^C}{\partial(1-\tau_L)}\frac{1-\tau_L}{M}.$$

(AII-3)

Substituting (AII-3) and the household budget constraint (9) in (AII-1) after some manipulation gives;

$$\frac{\partial l^c}{\partial \tau_L}=\frac{\partial X^c}{\partial(1-\tau_L)}\frac{L+S}{X}. \tag{AII-4}$$

Since the change in the price of good $X$ is equal to the change in the tax rate, the Slutsky equations give:

$$\frac{\partial l}{\partial \tau_X}=\frac{\partial l^c}{\partial \tau_X}-\frac{\partial l}{\partial I}X\ , \tag{AII-5}$$

$$\frac{\partial l}{\partial \tau_L}=\frac{\partial l^c}{\partial \tau_L}-\frac{\partial l}{\partial I}(L+S). \tag{AII-6}$$

Using (AII-4) in (AII-6) can be written as;

$$\frac{\partial l}{\partial I}=\frac{\partial X^c}{\partial(1-\tau_L)}\frac{1}{X}-\frac{\partial l}{\partial \tau_L}\frac{1}{(L+S)}. \tag{AII-6a}$$

The expression for the (cost-side) tax interaction effect from equation (16) becomes;

$$dW^I=-\eta\left[\tau_L\frac{\partial l}{\partial \tau_X}+\left(\tau_M+\frac{\partial S}{\partial H}\frac{\partial H}{\partial M}\right)\frac{dM}{d\tau_X}\right]. \tag{AII-7}$$

Using (AII-5) in (AII-7) we get;

$$dW^I=-\eta\left[\tau_L\left(\frac{\partial l^c}{\partial \tau_X}-\frac{\partial l}{\partial I}X\right)+\left(\tau_M+\frac{\partial S}{\partial H}\frac{\partial H}{\partial M}\right)\frac{dM}{d\tau_X}\right].` \tag{AII-8}$$

Again the Slutsky symmetry property is as follows:

$$\frac{\partial l^c}{\partial \tau_X}=-\frac{\partial X^c}{\partial(1-\tau_L)}. \tag{AII-9}$$

Using (AII-9) and (AII-6a) in (AII-8) can be expressed as;

$$dW^{I} = -\eta\left[\tau_L\left(\frac{\partial l^c}{\partial \tau_X} - \frac{\partial l}{\partial I}X\right) + \left(\tau_M + \frac{\partial S}{\partial H}\frac{\partial H}{\partial M}\right)\frac{dM}{d\tau_X}\right]$$

$$= -\eta\tau_L \frac{X}{L+S}\frac{\partial l}{\partial \tau_L} - \eta\left(\tau_M + \frac{\partial S}{\partial H}\frac{\partial H}{\partial M}\right)\frac{dM}{d\tau_X}$$

$$= (1-\eta)X - \eta\left(\tau_M + \frac{\partial S}{\partial H}\frac{\partial H}{\partial M}\right)\frac{dM}{d\tau_X}. \qquad \text{(AII-10)}$$

From (16) the benefit-side tax interaction effects $dW^{IB}$ is:

$$dW^{IB} = -\left(\frac{\partial S}{\partial H}\frac{\partial H}{\partial Q} + \eta\tau_L\frac{\partial l}{\partial Q}\right)\frac{dQ}{d\tau_X}. \qquad \text{(AII-11)}$$

By the partial derivative of the household budget constraint (9), the change in spending on *X*, *Y* and *l* for a change in *Q* becomes as follows:

$$(1-\tau_L)\frac{\partial l}{\partial Q} + (1+\tau_X)\frac{\partial X}{\partial Q} + \frac{\partial Y}{\partial Q} = -(1-\tau_M)\frac{\partial M}{\partial Q}. \qquad \text{(AII-12)}$$

Similarly as like (AII-12) for the change in *I*, (9) becomes:

$$(1-\tau_L)\frac{\partial l}{\partial I} + (1+\tau_X)\frac{\partial X}{\partial I} + \frac{\partial Y}{\partial I} = 1-(1-\tau_M)\frac{\partial M}{\partial I}. \qquad \text{(AII-13)}$$

Weak separability of the health in the utility function implies that leisure demand is determined only by the relative prices of *l*, *X* and *Y*, and by total spending on those goods.

Those relative prices are not affected by changes in *Q* or changes in *I.* As a result, the derivative of *l* with respect to *Q* will equal the derivative of *l* with respect to *I* times the ratio of the derivative of spending on those goods with respect to *Q* (AII-12) to the derivative of spending on those goods with respect to *I* (AII-13). Hence (AII-13) becomes;

$$\frac{\partial l}{\partial Q} = -\frac{\partial l}{\partial I}\left[1-(1-\tau_M)\frac{\partial M}{\partial I}\right]^{-1}(1-\tau_M)\frac{\partial M}{\partial Q}. \qquad \text{(AII-14)}$$

Again the uncompensated labor supply elasticity is;

$$\in_L = \frac{\partial(L+S)}{\partial(1-\tau_L)}\frac{1-\tau_L}{L+S} = \frac{\partial l}{\partial \tau_L}\frac{1-\tau_L}{L+S} = \frac{\eta-1}{\eta\tau_L}(1-\tau_L), \qquad \text{(AII-15)}$$

and the income elasticity of labor supply is;

$$\in_{LI} = \frac{\partial(L+S)}{\partial I}\frac{(1-\tau_L)(L+S)}{L+S} = -\frac{\partial l}{\partial I}(1-\tau_L). \qquad \text{(AII-16)}$$

Dividing (AII-16) by (AII-15) we get;

$$\eta\tau_L\frac{\partial l}{\partial I} = -(\eta-1)\frac{\in_{LI}}{\in_L}. \qquad \text{(AII-17)}$$

Multiplying (AII-14) by $\eta\tau_L$ and using (AII-17) we get;

$$\eta\tau_L\frac{\partial l}{\partial Q} = -\eta\tau_L\frac{\partial l}{\partial I}\left[1-(1-\tau_M)\frac{\partial M}{\partial I}\right]^{-1}(1-\tau_M)\frac{\partial M}{\partial Q}$$

$$= -(\eta-1)\frac{\in_{LI}}{\in_L}\left[1-(1-\tau_M)\frac{\partial M}{\partial I}\right]^{-1}(1-\tau_M)\frac{\partial M}{\partial Q}. \qquad \text{(AII-18)}$$

Now using (AII-18) in (AII-11) we find that;

$$dW^{IB} = \left\{-\frac{\partial S}{\partial H}\frac{\partial H}{\partial Q} - (\eta-1)\frac{\in_{LI}}{\in_L}\left[1-(1-\tau_M)\frac{\partial M}{\partial I}\right]^{-1}(1-\tau_M)\frac{\partial M}{\partial Q}\right\}\frac{dQ}{d\tau_X}. \qquad \text{(AII-19)}$$

Finally using the value of $dW^I$ and $dW^{IB}$ from (AII-10) and (AII-18) in (16) we get;

$$\frac{1}{\lambda}\frac{dU}{d\tau_X} = (\tau_X-\tau_P)\frac{dX}{d\tau_X} + (\eta-1)\left(X+\tau_X\frac{dX}{d\tau_X}\right) - (1-\eta)X - \eta\left(\tau_M + \frac{\partial S}{\partial H}\frac{\partial H}{\partial M}\right)\frac{dM}{d\tau_X}$$

$$+\left\{-\frac{\partial S}{\partial H}\frac{\partial H}{\partial Q} - (\eta-1)\frac{\in_{LI}}{\in_L}\left[1-(1-\tau_M)\frac{\partial M}{\partial I}\right]^{-1}(1-\tau_M)\frac{\partial M}{\partial Q}\right\}\frac{dQ}{d\tau_X},$$

$$\frac{1}{\lambda}\frac{dU}{d\tau_X} = (\eta\tau_X-\tau_P)\frac{dX}{d\tau_X} - \eta\left(\tau_M + \frac{\partial S}{\partial H}\frac{\partial H}{\partial M}\right)\frac{dM}{d\tau_X} + \left\{-\frac{\partial S}{\partial H}\frac{\partial H}{\partial Q}\right.$$

$$-(\eta-1)\frac{\in_{LI}}{\in_L}\left[1-(1-\tau_M)\frac{\partial M}{\partial I}\right]^{-1}(1-\tau_M)\frac{\partial M}{\partial Q}\Bigg\}\frac{dQ}{d\tau_X}. \qquad \text{(AII-20)}$$

This is the required equation (17).