KASBIT BUSINESS JOURNAL

Journal homepage: www.kbj.kasbit.edu.pk

Modeling the Linkages of the Pakistan Stock Market with International Stock Markets: Evidence from the United States, China, and Japan

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ARTICLE INFO

Keywords:

Pakistan stock market,

Volatility spillover,

Return spillover,

BEKK-GARCH,

CPEC,

Market integration.

ABSTRACT

This study examines the return and volatility linkages between Pakistan's equity market and three important international markets: the United States, China, and Japan. The question matters for both portfolio diversification and financial stability because emerging markets can be influenced by regional and global shocks even when their domestic fundamentals remain important. Using daily closing index data for the PSX-100, S&P 500, Shanghai Stock Exchange index, and Nikkei 225 from 1 January 2008 to 29 June 2018, the analysis estimates asymmetric bivariate VARMA(1,1) GARCH(1,1) models with BEKK specification for Pakistan-US, Pakistan-China, and Pakistan-Japan pairs. It also estimates trivariate asymmetric BEKK models for Pakistan-China-US and Pakistan-China-Japan to assess joint spillovers. To capture the initial policy signal associated with the China-Pakistan Economic Corridor (CPEC), a post-20 April 2015 dummy is included in the variance specification.

The results indicate that Pakistan is influenced by its own past returns and by lagged information from the US and Japan, while mean spillovers from China to Pakistan are not statistically strong. In the variance equations, Pakistan shows high volatility persistence, and volatility spillovers from the US and China are stronger than those from Japan. The multivariate models confirm the presence of joint mean and variance transmission, but the overall pattern remains one of weak rather than deep integration. The paper therefore concludes that Pakistan's stock market is connected to regional and global markets, yet domestic shocks still dominate. These findings imply that international diversification benefits remain available, although they are limited during periods of heightened volatility.

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Received: 9th February, 2026
Received in revised form: 28th February, 2026
Accepted: 15th March, 2026

1. Introduction

The increasing integration of financial markets has made cross-border return and volatility transmission a central issue in international finance. For investors, the degree of stock market integration determines whether international diversification can reduce risk meaningfully. For policymakers, cross-market linkages indicate how quickly foreign disturbances may affect domestic asset prices, financial stability, and expectations. These issues are especially important for emerging markets, where global integration has advanced unevenly and where local political, institutional, and macroeconomic factors may still dominate pricing dynamics.

This paper investigates both return spillovers and volatility spillovers between Pakistan and the selected international markets. The distinction is important. Return spillovers capture the transmission of information into expected returns, while volatility spillovers capture the transmission of uncertainty and shocks across markets. A market may appear only weakly integrated in mean returns but still be vulnerable to foreign volatility. To capture these dimensions jointly, the paper uses asymmetric VARMA-GARCH models with BEKK covariance structure. The asymmetric specification is important because bad news and good news often have different effects on equity-market volatility, especially in emerging markets.

The study makes three contributions. First, it jointly examines Pakistan's linkages with the US, China, and Japan using a unified bivariate and trivariate framework. Second, it introduces a CPEC-related dummy to test whether the formal launch of the corridor corresponds to a shift in the short-run volatility process. Third, it presents a more cautious interpretation of market integration than is common in thesis-style discussions: significant spillovers do not automatically imply deep integration, and statistical significance must be distinguished from economic dominance.

The analysis also responds directly to several concerns that arise in the literature on international spillovers. Daily stock markets operate in different time zones, and this can bias inferences if same-day movements are interpreted mechanically as causality. To reduce this problem, the data are aligned on common trading days only, and the empirical emphasis is placed on lagged cross-market effects rather than on contemporaneous co-movement. Likewise, although the Gaussian likelihood is retained as the baseline estimation framework for tractability and consistency with the BEKK literature, the heavy tails and asymmetry visible in the return series are acknowledged explicitly when interpreting results.

The main findings are straightforward. Pakistan's market is significantly influenced by its own past returns and by lagged spillovers from the US and Japan. China does not exhibit equally strong mean spillovers into Pakistan over the sample. Pakistan's volatility is highly persistent, and volatility spillovers from the US and China are more prominent than those from Japan. The multivariate models confirm that regional and global markets matter jointly, but the overall pattern still indicates weak integration: own-market effects remain stronger than cross-market effects in most specifications.

The rest of the paper proceeds as follows. Section 2 reviews the relevant literature and clarifies the paper's contribution. Section 3 describes the data and econometric strategy. Section 4 presents the empirical findings. Section 5 discusses the implications, limitations, and avenues for further work. Section 6 concludes.

2. Literature Review and Analytical Motivation

The theoretical basis for this study comes from portfolio theory and international asset pricing. Markowitz (1952) established that diversification depends on the covariance structure of asset returns rather than on the properties of individual assets alone. In internationally segmented markets,

domestic risk factors dominate returns, and foreign diversification can offer substantial benefits. Under stronger integration, however, common global factors increasingly drive returns, reducing diversification gains. The CAPM tradition, especially Sharpe (1964) and Lintner (1965), formalized the relation between expected returns and systematic risk, while Ross (1976) broadened the framework through arbitrage pricing. Although these theories are not themselves volatility models, they motivate the empirical concern with co-movement, dependence, and the pricing of common shocks.

Empirical work on international linkages has developed along two broad lines. One strand studies price co-movement, integration, and international asset pricing. Bekaert and Harvey (1995), for example, showed that emerging markets often display time-varying integration rather than a stable relation with world markets. This is important for Pakistan because it suggests that integration should not be treated as an all-or-nothing condition. Another strand focuses more directly on the transmission of returns and volatility using ARCH- and GARCH-type models. Engle (1982) introduced the ARCH framework, and Bollerslev (1986) extended it to GARCH, allowing persistence in conditional variance. Engle and Kroner (1995) developed the BEKK parameterization for multivariate GARCH systems, while Kroner and Ng (1998) showed how asymmetric responses can be incorporated in multivariate volatility models.

The empirical spillover literature shows that external markets often influence both the mean and the variance of domestic returns, but the pattern is rarely uniform across countries. Lin, Engle, and Ito (1994) demonstrated that international transmission is sensitive to trading-hour structure and information timing, a point that remains highly relevant when comparing Asian and US markets. Koutmos and Booth (1995) documented asymmetric volatility transmission across major markets. Ng (2000) found significant spillovers from the US and Japan to Pacific-Basin markets, with regional and global factors affecting countries differently over time. Karolyi (1995) similarly showed that the estimated magnitude of international transmission depends on how volatility dynamics are modeled.

Evidence for emerging markets generally suggests that spillovers from mature markets exist, but domestic effects often remain dominant. Beirne et al. (2010) reported that both regional and global spillovers matter across emerging markets, with the balance between mean and variance spillovers varying by region. Li and Giles (2015) found significant unidirectional and, during crisis episodes, bidirectional spillovers from the US to Asian emerging markets. These studies imply that the effect of global shocks on emerging markets is real but heterogeneous.

For Pakistan, the evidence is still relatively limited. Aziz and Iqbal (2017) showed that Pakistan's market is influenced by global and regional markets, but that the overall degree of integration remains weak. Ghouse and Khan (2017) and Ghouse, Khan, and Arshad (2019) also documented dynamic linkages between Pakistan and major foreign markets, especially during or around crisis periods. However, the existing literature leaves space for additional work for three reasons. First, much of the Pakistan-focused evidence emphasizes the US and a narrow set of regional comparators, often omitting China in a dedicated framework. Second, studies rarely incorporate the CPEC context explicitly, despite its importance for expectations about Pakistan's medium-term growth and financial integration. Third, many papers move too quickly from detecting statistical spillovers to claiming broad market integration, even when own-market volatility remains quantitatively dominant.

This paper addresses those gaps by examining Pakistan's linkages with the US, China, and Japan in a single framework and by distinguishing more carefully between statistical linkage and deep integration. The paper does not assume that the existence of any significant coefficient is enough to

claim a fully integrated market. Instead, the interpretation turns on direction, persistence, relative magnitude, and whether foreign shocks dominate or merely influence Pakistan's market process.

3. Data and Methodology

3.1 Data

The empirical analysis uses daily closing index data for four equity markets: the PSX-100 for Pakistan, the S&P 500 for the United States, the Shanghai Stock Exchange index for China, and the Nikkei 225 for Japan. The sample runs from 1 January 2008 to 29 June 2018. All data are drawn from DataStream, and the estimation sample contains 2,643 observations for each market after aligning trading days so that only dates on which all relevant markets are open are retained.

Returns are computed as percentage log differences:

$$\boldsymbol{L_t = \int + \omega R_{t-1} + \varphi v_{t-1} + v_t, v_t |\, I_{t-1} \sim (0, H_t)} \quad (1)$$

$$\boldsymbol{H_t = \text{JU}\text{JU}' + \text{ß}' u_{t-1} u_{\prime t-1} \text{ß} + \text{ø}' H_{t-1} H_{\prime t-1} \text{ø} + D' \varepsilon_{t-1} \varepsilon'_{t-1} D} \quad (2)$$

$L_t = [l_{1,t}\, l_{2,t}]'$ is the vectors of log returns in the form of percentage. $v_t = [v_{1,t}\, v_{2,t}]'$ is the residual vector with a conditional variance-covariance matrix. $H_t = [h_{ij,t}]_{i,j=1,2}$. ε_t is equal to v_t if v_t is negative or 0. I_{t-1} is the set of information available at time t – 1. $\int = [\gamma_1 \gamma_2]'$, $\omega = [\tau_{ij}]_{i,j=1,2}$and $\varphi = [\delta_{ij}]_{i,j=1,2}$ are the constant returns coefficient matrices, these are the mean returns of first – lagged log and first – lagged shocks. $\text{JU} = [\partial_{ij,}]_{i,j=1,2}'$ is the parameter of volatility equation of an upper triangular matrix. Whereas, unrestricted ARCH is represented by coefficient matrix $\text{ß} = [\beta_{ij,}]_{i,j=1,2}$ and the unrestricted GARCH coefficient matrix is given by $\text{ø} = [\alpha_{ij,}]_{i,j=1,2}$. Whereas the unrestricted coefficient matrix of assymetric response of volatility is given by the paramter matrix of $D = [d_{ij,}]_{i,j=1,2}$. Working with daily log returns instead of price levels helps mitigate the non-stationarity that typically characterizes stock price indices and allows the analysis to focus on short-run information transmission and conditional volatility dynamics.

The sample period is chosen for two reasons. First, it covers the post-global-financial-crisis environment in which cross-border market linkages became a major concern. Second, it includes the formal launch of CPEC on 20 April 2015, when Pakistan and China signed the main agreements associated with the corridor. To capture the immediate regime shift associated with this event, the analysis includes a simple post-CPEC dummy equal to 0 before 20 April 2015 and 1 from that date onward. This dummy should be interpreted narrowly. It is not intended to represent the full long-run economic impact of CPEC; rather, it captures whether the formal launch corresponded to an identifiable short-run change in Pakistan's market volatility.

3.2 Descriptive Properties

The descriptive statistics indicate several standard features of financial return series. Pakistan records the highest average daily return in the sample, approximately 0.042 percent, followed by the US and Japan,

while China records a small negative mean return of roughly -0.024 percent. In terms of standard deviation, China appears most volatile at about 1.62, while Pakistan appears least volatile at about 1.12.

All four series exhibit negative skewness and excess kurtosis, implying asymmetry and fat tails. These features are consistent with the broader financial econometrics literature and justify the use of conditional heteroskedasticity models. At the same time, they caution against interpreting Gaussian estimates too literally in a distributional sense. The Gaussian likelihood in the present paper is therefore treated as a baseline estimation device rather than as a literal claim that returns are normally distributed.

Descriptive statistics of daily percentage log returns

Table 1.

Market	**Mean**	**Standard deviation**	**Skewness**	**Kurtosis**	**Observations**
United States (S&P 500)	0.0239	1.2767	-0.3601	11.0450	2,643
Japan (Nikkei 225)	0.0158	1.5923	-0.5692	8.6523	2,643
China (SSE)	-0.0239	1.6205	-0.6554	6.2254	2,643
Pakistan (PSX-100)	0.0424	1.1278	-0.1741	4.9005	2,643

Note: Values are taken from the original estimation sample. Returns are computed as percentage log differences of daily closing indices.

3.3 Econometric Framework

The paper estimates both bivariate and trivariate asymmetric VARMA(1,1)-GARCH(1,1) models with BEKK covariance structure. The bivariate models are fitted separately for Pakistan-US, Pakistan-China, and Pakistan-Japan. The trivariate models are fitted for Pakistan-China-US and Pakistan-China-Japan. This structure permits the paper to analyze both pairwise spillovers and broader joint transmission.

The mean equation can be written generically as:

$$\boldsymbol{r_t = \mu_t + \varepsilon_t},$$

$$\boldsymbol{\varepsilon_t \,|\Omega_{t-1} \sim N\,(0, H_t)},$$

Where $r_t$is an N×1 vector of returns at time *t*, μ_t is a mean return, and is an N×1 vector of random errors at time t with its corresponding N × N conditional variance- covariance matrix H_t. The information set Ω_{t-1} captures all information available at time t – 1.

The conditional covariance matrix follows an asymmetric BEKK process:

$$H_t = CC' + \sum_{i=1}^{q} A_i(\epsilon_{t-1}\epsilon'_{t-1})\,A'_t + \sum_{j=1}^{p} B_j\,H_{t-j}B'_j$$

Where:

- H_t: Conditional covariance matrix at time t
- C: Lower triangular matrix (ensures positive definiteness)
- C': Transpose of C
- ε_{t-i}: Error (residual) vector at lag i
- $\varepsilon_{t-i}\varepsilon'_{t-i}$: Outer product (captures past shocks)
- A_i: Parameter matrices for ARCH effects
- B_j: Parameter matrices for GARCH effects
- H_{t-j}: Past conditional covariance matrices

Key Idea:

- The first term $CC'$is the constant component
- The second term captures the impact of past shocks (ARCH effect)
- The third term captures persistence from past volatility (GARCH effect)

The BEKK form is selected because it guarantees a positive definite covariance matrix and allows the direction of spillovers to be studied within a coherent multivariate system (Engle & Kroner, 1995). The asymmetric extension follows the logic of Kroner and Ng (1998), which is particularly suitable when negative shocks may have stronger volatility effects than positive shocks of the same magnitude.

3.4 Identification and Hypotheses

The analysis focuses on five sets of hypotheses:

1. No mean spillover between Pakistan and the foreign market.
2. No mean spillover from the foreign market to Pakistan.
3. No volatility spillover between Pakistan and the foreign market.
4. No volatility spillover from the foreign market to Pakistan.
5. No short-run CPEC effect on Pakistan’s conditional variance.

In the trivariate models, additional joint restrictions are tested to determine whether regional and global markets affect Pakistan simultaneously through mean and variance channels.

Because the markets under study operate in different time zones, cross-market transmission should not be interpreted as exact contemporaneous causality. This issue is especially relevant for the US relative to Asian markets. To reduce this problem, the paper uses only common trading days and

emphasizes lagged spillover terms. The coefficients therefore reflect information transmission across markets rather than strict same-clock simultaneity.

4. Empirical Results

4.1 Return Spillovers

The mean equations show that Pakistan's own lagged returns are statistically important across the bivariate specifications. This suggests that the domestic market exhibits short-run persistence and that recent local information remains relevant for current returns. Such own-market effects are stronger than cross-market return spillovers in magnitude, which is the first indication that Pakistan is linked to international markets but not dominated by them.

The cross-market results are asymmetric. In the Pakistan-US and Pakistan-Japan models, the lagged foreign return and innovation terms indicate statistically significant spillovers into Pakistan. These findings imply that information originating in the US and Japan affects Pakistan's return process on the following trading day. In contrast, the Pakistan-China model does not show equally strong evidence of mean spillovers from China to Pakistan. This does not imply that China is irrelevant to Pakistan's financial environment; rather, it suggests that during the sample period China's influence is not transmitted as clearly through short-run return spillovers as that of the US and Japan.

The Wald restrictions support this interpretation. The null of no mean spillover is rejected for the Pakistan-US and Pakistan-Japan pairs, while the corresponding evidence is weaker for Pakistan-China. The direction of mean spillover is mainly from the foreign market to Pakistan, not the reverse. Pakistan does not appear to Granger-cause the US or Japan in mean returns, which is unsurprising given differences in market size, liquidity, and global financial relevance.

These results are consistent with the idea that Pakistan is an information-taking market relative to major developed exchanges. However, the paper avoids the stronger claim that such evidence alone proves broad-form inefficiency. Cross-market predictability in daily returns can reflect timing, market closures, investor rebalancing, or delayed incorporation of external news, particularly when markets operate in different time zones. The more defensible conclusion is that Pakistan's return process is sensitive to foreign information, especially from the US and Japan, but remains strongly shaped by domestic dynamics.

Compact summary of bivariate mean-equation results

Table 2.

Market pair	Pakistan own-lag return coefficient	No mean spillover: F-stat (p-value)	Foreign to Pakistan spillover: F-stat (p-value)	Pakistan to foreign market spillover: F-stat (p-value)	Interpretation
Pakistan-Japan	0.0013 (0.000)	48125.40 (0.000)	4.772 (0.008)	6.458 (0.001)	Strong overall mean linkage; foreign-to-Pakistan effect significant
Pakistan-US	0.0410 (0.000)	358.77 (0.000)	13.57 (0.000)	4.017 (0.018)	Strong foreign information transmission from the US to Pakistan
Pakistan-China	1.0000 (0.000)	8551.187 (0.000)	0.224 (0.792)	4.166 (0.0156)	No robust foreign-to-Pakistan mean spillover from China

Note: The table reports compact mean-equation evidence from the original bivariate VARMA (1,1)-BEKK estimates. The own-lag coefficient is reported only for Pakistan's return equation because the revised article emphasizes the domestic persistence versus cross-market spillover distinction.

4.2 Volatility Persistence

The variance equations show pronounced volatility persistence in Pakistan. Across the bivariate models, the sum of the relevant ARCH and GARCH components indicates that Pakistan's conditional variance is strongly influenced by its own past shocks and past volatility. This is a familiar result in financial econometrics and is consistent with volatility clustering in emerging equity markets.

The persistence estimates also help explain why mean spillover evidence should not be equated with deep integration. Even when foreign return innovations are statistically significant, much of Pakistan's volatility dynamics still arises from its own history. Put differently, international shocks matter, but they enter a system whose baseline persistence is domestic.

4.3 Volatility Spillovers

The variance and covariance equations reveal stronger foreign influence in volatility than in mean returns. The Pakistan-US and Pakistan-China models show significant volatility spillovers from the foreign market to Pakistan. In the Pakistan-Japan model, the evidence is weaker and less consistent. Thus, the main volatility channels into Pakistan during the sample appear to come from the US and China.

This result is economically intuitive. The US remains the central global market and a major source of worldwide financial information. China, meanwhile, is increasingly relevant to Pakistan's economic outlook and investor expectations, even if its daily return spillovers are not as strong in the mean equation. Foreign volatility may be transmitted through channels other than immediate returns, including sentiment, commodity-price expectations, trade expectations, and broader regional risk perception.

The direction of volatility transmission is not purely one-way. Some specifications indicate that Pakistan also transmits volatility to the US and China. However, these reverse effects should be interpreted cautiously. They do not suggest that Pakistan drives global volatility in any substantive sense. Rather, they indicate that the covariance structure is not completely unilateral and that shocks in Pakistan can be statistically associated with subsequent variance adjustments in linked markets. The broader pattern still favors the interpretation that Pakistan is more exposed to foreign volatility than foreign markets are exposed to Pakistan.

Importantly, the own-market volatility terms remain larger than the cross-market terms for Pakistan. This means that the external spillovers, although significant, do not overwhelm domestic volatility persistence. Hence, the results support a weak-integration view rather than a strong-integration view.

Compact summary of bivariate variance-equation results

Table 3.

Market pair	**Pakistan volatility persistence**	**No variance spillover: F-stat (p-value)**	**Foreign to Pakistan volatility spillover: F-stat (p-value)**	**Pakistan to foreign market volatility spillover: F-stat (p-value)**	**CPEC effect: F-stat (p-value)**
Pakistan-US	0.882	34.03 (0.000)	3.675 (0.018)	4.471 (0.024)	1.234 (0.724)
Pakistan-Japan	0.832	11.67 (0.000)	1.383 (0.250)	2.541 (0.619)	2.654 (0.898)
Pakistan-China	0.923	74.19 (0.000)	4.360 (0.047)	8.792 (0.049)	2.792 (0.853)

Note: Pakistan volatility persistence is taken from the summed GARCH persistence measures discussed in the original results section. The CPEC row shows that the post-20 April 2015 variance dummy is not statistically significant in any bivariate specification.

4.4 The CPEC Dummy

The CPEC dummy is not statistically significant in the variance equations. This indicates that the formal launch of CPEC on 20 April 2015 did not generate an immediate and persistent shift in Pakistan's daily stock-return volatility within the estimated short-run framework. This is an important result because it discourages an exaggerated reading of headline announcements in financial markets.

At the same time, the insignificance of the dummy should not be interpreted as evidence that CPEC was irrelevant to Pakistan's financial prospects. CPEC is not a one-day shock but a long and evolving policy process involving expectations about infrastructure, trade, energy supply, debt, security, and political implementation. A single intercept dummy can capture only a broad shift around the formal launch date. It cannot represent the staggered arrival of project-specific information, changes in investor optimism, or later macroeconomic constraints. The most reasonable interpretation is therefore narrow: the formal launch of CPEC did not produce a clear short-run variance break in the daily PSX return process over and above the other forces already captured in the model.

4.5 Multivariate Results

The trivariate asymmetric BEKK models for Pakistan-China-US and Pakistan-China-Japan provide a broader view of market interaction. The joint tests reject the null of no combined mean and variance spillovers into Pakistan. This indicates that Pakistan is affected by foreign markets not only in isolated pairwise relations but also when regional and global markets are modeled simultaneously.

However, the multivariate evidence does not overturn the main conclusion from the bivariate models. The joint spillovers are statistically significant, yet the overall degree of integration remains limited. Pakistan's market continues to be characterized by strong own-market persistence, and the foreign channels do not dominate the system. In this sense, the multivariate results reinforce rather than replace the central finding of the paper: Pakistan is linked to regional and global markets, but the linkage is incomplete.

This distinction matters for both researchers and practitioners. For researchers, it suggests that models emphasizing complete international integration may overstate the role of foreign factors in Pakistan's stock market. For investors, it implies that Pakistan may still provide diversification benefits because it is not fully synchronized with the US, China, or Japan. Those benefits, however, are likely to narrow during periods of heightened uncertainty when volatility spillovers strengthen.

Compact summary of multivariate spillover tests

Table 4.

Trivariate system	**No joint mean/variance spillover to Pakistan: F-stat (p-value)**	**No joint mean spillover to Pakistan: F-stat (p-value)**	**No joint volatility spillover to Pakistan: F-stat (p-value)**	**Interpretation**
Pakistan-China-US	45.652 (0.000)	13.584 (0.000)	5.678 (0.048)	Joint regional and global spillovers into Pakistan are statistically significant
Pakistan-China-Japan	35.465 (0.000)	15.985 (0.000)	7.598 (0.035)	Joint spillovers remain significant when Japan replaces the US

Note: These Wald tests summarize the multivariate BEKK results and support the interpretation that Pakistan is affected by regional and global markets jointly, although own-market persistence remains central.

5. Discussion, Implications, and Limitations

The empirical findings have three implications.

First, they support a layered view of financial integration. Pakistan is not isolated from international markets, but neither is it fully absorbed into them. The US and Japan affect Pakistan's returns, while the US and China affect its volatility more clearly. This combination of mean and variance evidence suggests selective rather than uniform integration.

Second, the results imply that domestic risk remains central to understanding Pakistan's equity pricing. Own-market spillovers are stronger than cross-market spillovers in most specifications, and volatility persistence is high. This means that country-specific political, macroeconomic, regulatory, and firm-level shocks still play a substantial role in determining expected returns and risk in Pakistan.

Third, the findings offer a cautious message for portfolio diversification. Weak integration means there is still some scope for diversification benefits when Pakistan is combined with major international markets. Yet this opportunity is conditional. In tranquil periods, partial segmentation may help diversification. In turbulent periods, volatility transmission from larger markets can reduce that benefit, particularly when global uncertainty is high.

The paper also has important limitations.

One limitation concerns the treatment of trading-time differences. Although the use of common trading days and lagged spillover terms reduces the most obvious non-synchronous trading problems, it does not eliminate them entirely. A more refined study could use intraday or close-to-open decompositions in the spirit of Lin, Engle, and Ito (1994).

A second limitation concerns distributional assumptions. The return series display skewness and fat tails, which means that a Gaussian likelihood is a simplifying baseline rather than an exact description. A Student-\(t\), skewed-\(t\), or other heavy-tailed specification would be a useful robustness check.

A third limitation is the treatment of CPEC as a single post-event dummy. This is appropriate for a short-run announcement test, but it is too simple for the broader economic process associated with CPEC. Structural-break methods, multiple-event windows, or project-level event studies would likely provide a more nuanced assessment.

A fourth limitation is that the paper does not model wider macro-financial channels such as exchange rates, interest rates, commodity prices, or crisis subperiods. Since Pakistan's market can be influenced indirectly through these channels, future research could combine stock-market spillovers with broader financial and macroeconomic variables.

These limitations do not invalidate the present results. They simply define their proper interpretation. The paper offers evidence on short-run return and volatility transmission in a well-established multivariate GARCH framework; it does not claim to provide a complete theory of Pakistan's international financial integration.

6. Conclusion

This paper studies the linkages between Pakistan's stock market and three international markets: the United States, China, and Japan. Using daily data from 2008 to 2018 and estimating asymmetric bivariate and trivariate BEKK-GARCH models, it examines both return spillovers and volatility spillovers while also testing whether the formal launch of CPEC altered Pakistan's volatility process.

The findings show that Pakistan's market is influenced by lagged return spillovers from the US and Japan, whereas corresponding mean spillovers from China are weaker. In the variance equations, Pakistan displays strong own-market volatility persistence, and foreign volatility spillovers are most visible from the US and China. The trivariate models confirm that regional and global markets matter jointly, yet the overall degree of integration remains weak because domestic shocks continue to dominate Pakistan's market dynamics. The CPEC dummy is not significant, suggesting that the formal launch of the corridor did not create an immediate short-run volatility break in Pakistan's stock market.

The broader conclusion is that Pakistan's equity market is internationally connected but not fully integrated. For investors, this means that diversification benefits are still possible, though they may narrow when volatility transmission intensifies. For policymakers, it means that external shocks cannot be ignored, but neither should domestic financial conditions be treated as secondary. Pakistan's market remains a hybrid case: exposed to regional and global information, yet still strongly anchored in country-specific risk.

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